

# Anisotropic thermal conductivity of magnetic fluids

Xiaopeng Fang, Yimin Xuan\*, Qiang Li

*School of Power Engineering, Nanjing University of Science & Technology, Nanjing 210094, China*

Received 18 February 2008; received in revised form 3 April 2008; accepted 23 June 2008

## Abstract

Considering the forces acting on the particles and the motion of the particles, this study uses a numerical simulation to investigate the three-dimensional microstructure of the magnetic fluids in the presence of an external magnetic field. A method is proposed for predicting the anisotropic thermal conductivity of magnetic fluids. By introducing an anisotropic structure parameter which characterizes the non-uniform distribution of particles suspended in the magnetic fluids, the traditional Maxwell formula is modified and extended to calculate anisotropic thermal conductivity of the magnetic fluids. The results show that in the presence of an external magnetic field the magnetic nanoparticles form chainlike clusters along the direction of the external magnetic field, which leads to the fact that the thermal conductivity of the magnetic fluid along the chain direction is bigger than that along other directions. The thermal conductivity of the magnetic fluids presents an anisotropic feature. With the increase of the magnetic field strength the chainlike clusters in the magnetic fluid appear to be more obvious, so that the anisotropic feature of heat conduction in the fluids becomes more evident.

© 2008 National Natural Science Foundation of China and Chinese Academy of Sciences. Published by Elsevier Limited and Science in China Press. All rights reserved.

**Keywords:** Magnetic fluids; Anisotropic thermal conductivity; Maxwell formula

## 1. Introduction

Magnetic fluids are colloidal suspensions containing surfactant-coated magnetic nanoparticles dispersed in carrier liquids such as oil and water. As functional fluids, they have attracted much attention over the past decades. So far, they have been applied in many fields such as mechanical engineering, bioengineering, thermal engineering, etc. [1–2]. The magnetic particles in the magnetic fluid can interact easily with an applied magnetic field, which in turn can influence the microstructure of the fluids. Experiments have shown that in the presence of an external magnetic field the magnetic particles tend to form chainlike clusters along the magnetic field direction. Many numerical simulation methods such as the Monte Carlo (MC) method, Brownian Dynamics (BD) method and Molecular Dynam-

ics (MD) method have been widely used for studying the microstructure of magnetic fluids under the influence of an external magnetic field. Akira et al. used MC method [3,4] and BD method [5] to investigate the microstructure of magnetic fluids in the presence of an external magnetic field. They compared the results obtained under different conditions. By taking into account the effects of non-spherical particle shape, Yoshihisa and his coworkers [6] studied the microstructure of magnetic fluids by the BD method. Li et al. [7] studied the microstructure of magnetic fluids in both absence and presence of an external magnetic field. They analyzed the dependence of the aggregation structure on both the particle–particle interaction strength and the magnetic field strength. All that research suggested that the magnetic particles tend to form chain-like clusters along the direction of the magnetic field. With increasing the magnetic field strength, the chainlike clusters become more obvious. Experimental reports [8,9] have shown that in the presence of an external magnetic field, the thermal conductivity of the magnetic fluids presents an anisotropic

\* Corresponding author. Tel.: +86 25 84315700; fax: +86 25 84315991.  
E-mail address: [ymxuan@mail.njust.edu.cn](mailto:yxmuan@mail.njust.edu.cn) (Y. Xuan).

feature. The thermal conductivity of magnetic fluids along the magnetic field direction is bigger than that along other directions. With increasing the magnetic field strength, the anisotropy of heat conduction in the magnetic fluid becomes more evident. The analysis above suggests that the anisotropic heat conduction in the magnetic fluid may result from the anisotropic microstructure of the magnetic fluid. Currently, the research concerning this is mainly limited to experimental methods. It is very necessary from a theoretical point of view to make clear the heat conduction mechanism in the magnetic fluids.

The purpose of this paper is to investigate the non-uniform distribution characteristic of particles in the magnetic fluids and the anisotropic heat conduction in the magnetic fluids. The microstructures of magnetic fluids in the presence of different external magnetic fields have been simulated to investigate the effect of the magnetic field on the aggregate structure of magnetic fluids. By taking into account the non-uniform distribution of particles in the magnetic fluids a prediction model for the thermal conductivity of magnetic fluids is proposed. Then the relationship between the microstructure and heat conduction in the magnetic fluid is analyzed. By modifying the particle volume fraction of the magnetic fluids, the traditional Maxwell formula is extended to calculate the anisotropic thermal conductivity of magnetic fluids.

## 2. Simulation of the microstructure of magnetic fluids

### 2.1. The model for motion

We assume that the magnetic particles are spherical and the surface is coated with a surfactant layer. Each particle is composed of a single magnetic domain. The interactions between particles are complicated. We take into account the dipolar interaction potential [10], the van der Waals attraction potential [11], the repulsion potential due to the overlapping of the surfactant layers [12] and the particle-field interaction potential [1]. According to these potentials, the forces and torques acting on the particles can be obtained [7].  $\mathbf{F}_{ij}^m$  denotes the force acting on the  $i$ th particle due to the dipolar interaction between two particles  $i$  and  $j$ ,  $\mathbf{F}_{ij}^v$  the van der Waals force,  $\mathbf{F}_{ij}^s$  the short range repulsive force due to surface surfactants,  $\mathbf{T}_{ij}^m$  the torque due to the dipolar interaction,  $\mathbf{T}_i^h$  the torque due to the external magnetic field,  $\mathbf{F}_i^B$  and  $\mathbf{T}_i^B$  are the random force and the random torque due to the collisions with solvent molecules [13]. The net external force  $\mathbf{F}_i = \sum_{j \neq i} (\mathbf{F}_{ij}^m + \mathbf{F}_{ij}^v + \mathbf{F}_{ij}^s) + \mathbf{F}_i^B$  drives the magnetic particle and the net torque  $\mathbf{T}_i = \sum_{j \neq i} \mathbf{T}_{ij}^m + \mathbf{T}_i^h + \mathbf{T}_i^B$  rotates the magnetic particle simultaneously. The Langevin equations for both translational and rotational motions are expressed as

$$m \frac{dv_i}{dt} + \xi_t v_i = F_i \quad (1)$$

$$I \frac{d\omega_i}{dt} + \xi_r \omega_i = T_i \quad (2)$$

where  $m$  and  $I$  are the mass and the moment of inertia respectively,  $\xi_t = 3\pi d\eta$  is the translational drag force coefficient,  $\xi_r = \pi d^3\eta$  the rotational drag force coefficient,  $d$  the diameter of the particle,  $\eta$  the viscosity of the base liquid,  $v_i$  the velocity of the particle, and  $\omega_i$  is the angular velocity of the particle.

Considering the force  $\mathbf{F}_i$  in Eq. (1) and  $\mathbf{T}_i$  in Eq. (2) as constants and supposing that a particle starts at the initial velocity  $v_i^0$  and the initial angular velocity  $\omega_i^0$ , the instantaneous velocity and angular velocity of the particle are obtained as

$$v_i = v_i^0 e^{-\frac{\xi_t}{m}t} + \frac{F_i}{\xi_t} (1 - e^{-\frac{\xi_t}{m}t}) \quad (3)$$

$$\omega_i = \omega_i^0 e^{-\frac{\xi_r}{I}t} + \frac{T_i}{\xi_r} (1 - e^{-\frac{\xi_r}{I}t}) \quad (4)$$

The equations of motion of  $\mathbf{r}_i$  and  $\mathbf{n}_i$  for interacting particles can be described as

$$\frac{dr_i}{dt} = v_i \quad (5)$$

$$\frac{dn_i}{dt} = \omega_i \times n_i \quad (6)$$

Suppose that we know the initial position  $\mathbf{r}_i^0$  and the initial magnetic moment  $\mathbf{n}_i^0$  of the particle. By integrating Eqs. (5) and (6), we can obtain the position and the dipole moment of the particle after a short time interval  $\Delta t$ .

$$r_i = r_i^0 + \frac{m}{\xi_t} \left( v_i^0 - \frac{F_i}{\xi_t} \right) (1 - e^{-\frac{\xi_t}{m}\Delta t}) + \frac{F_i}{\xi_t} \Delta t \quad (7)$$

$$n_i = n_i^0 + \left[ \frac{I}{\xi_r} \left( \omega_i^0 - \frac{T_i}{\xi_r} \right) (1 - e^{-\frac{\xi_r}{I}\Delta t}) + \frac{T_i}{\xi_r} \Delta t \right] \times n_i^0 \quad (8)$$

### 2.2. Results and discussion

We carry out the numerical simulations mentioned above to study the microstructures of magnetic fluids.  $N$  spherical particles of diameter of 20 nm are placed in a cubic cell with a side length of 22  $d$ . The thickness of the surfactant layer ( $\delta$ ) is 3 nm. Periodic boundary conditions are imposed in all spatial directions. The cut off distance of the interactive potentials is  $R_c = 200$  nm; the bulk magnetic moment of each particle is  $n = 2.0 \times 10^{-18}$  A m<sup>2</sup>; and the temperature is  $T = 293$  K. Non-dimensional parameters  $\lambda_h$  is defined as

$$\lambda_h = \frac{\mu_0 n h}{kT} \quad (9)$$

where  $\mu_0$  is the permeability,  $n$  the bulk magnetic moment,  $k$  the Boltzmann constant, and  $\lambda_h$  the energy of a magnetic dipole in the magnetic field relative to the thermal energy. As the temperature and the magnetic moment are both constant, a high value of  $\lambda_h$  indicates a strong magnetic field.

In the presence of different external magnetic fields, the microstructures of magnetic fluids with different particle volume fractions are simulated. The simulation starts at a

random particle distribution. After  $2 \times 10^5$  steps, the aggregation of particles reaches a steady state. Then, the magnetization of the magnetic fluid is obtained over another 50000 steps. The magnetization is a parameter which reflects the influence of the magnetic field on the tropism of particle's magnetic moment. It is defined as  $M = \frac{\sum \bar{n}_i}{V}$ , where  $\bar{n}_i$  is the component of magnetic moment  $n_i$  in the magnetic field direction; and  $V$  the volume of the simulation cell. When all the particles align their magnetic moments along the magnetic field direction, the magnetization reaches saturation  $M_s$ .

The simulation results show that without an external magnetic field the magnetic particles in the magnetic fluid aggregate to form clusters due to the particle–particle interaction, and when an external magnetic field is applied the magnetic particles tend to form chainlike clusters along the magnetic field direction. With the increase of field strength, the tendency of forming chain structures becomes more evident. For a relatively weak magnetic field, some short chains will be formed along the magnetic field. For a stronger magnetic field, much longer and straighter chain structures are formed along the magnetic field direction. Fig. 1 shows the microstructure of the magnetic fluid with particle volume fraction  $\phi = 1$  at  $\lambda_h = 5$ . The magnetization of the magnetic fluid can characterize the aggregation structure of the magnetic fluid. In the presence of an external magnetic field, the magnetic particles align their moments along the magnetic field direction. And the particles aggregate to form chainlike clusters. With the increase of the magnetic field strength, more particles align their moments along the magnetic field direction and the chainlike structure becomes more obvious.

Fig. 2 shows the magnetization curves of magnetic fluids with different volume fractions. It is obvious that the magnetization curves differ from each other. The differences are primarily due to the interparticle interactions. In a dilute magnetic fluid ( $\phi = 1$ ), the interparticle interactions are weak and its effect on the magnetization is negligible. In the presence of a relatively weak magnetic field ( $\lambda_h = 5$ ) the magnetization gets close to saturation and the chainlike structure of particles aggregates appears distinctly. With further increase of the magnetic field strength, the magne-

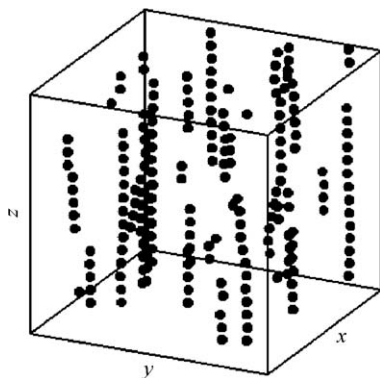


Fig. 1. Microstructure of magnetic fluid.

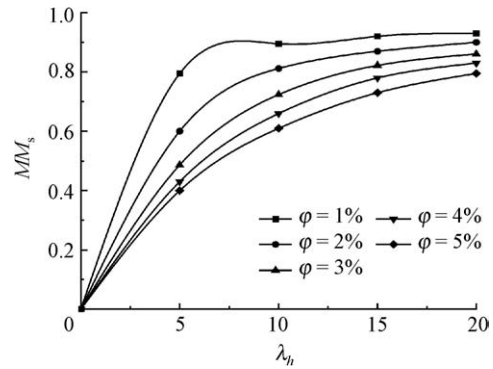


Fig. 2. Magnetization curves of magnetic field.

tization of the magnetic fluid increases very slowly and the aggregation structure of particles does not change much. For a large volume fraction  $\phi = 5$ , the strong interparticle interactions will restrain the magnetization of the magnetic fluid. Under the influence of a relatively weak field, the magnetization is small and the aggregation structure of particles in the magnetic fluid is mainly short chains. With increasing the magnetic field strength, the magnetization further increases and the chainlike structure in the magnetic fluid becomes more obvious.

### 3. Computation of thermal conductivity of magnetic fluid

When the magnetic dipole moment energy is stronger than the thermal energy, the magnetic particles tend to form chainlike clusters under the influence of an external magnetic field and the Brownian motion of the particles will be severely restricted due to the strong particle–particle interactions. This study aims at investigating the effect of aggregation structure on the heat conduction in the magnetic fluids, the model for the thermal conductivity of magnetic fluid is based on the microstructure of the magnetic fluids and the effect of particles' motion on the thermal conductivity is neglected.

#### 3.1. The model for thermal conductivity

The model of thermal transport in magnetic fluid is shown in Fig. 3. The cubic cell is filled with magnetic particles and base liquid. At the steady state, the equation for heat conduction in magnetic fluid can be expressed as

$$\frac{\partial}{\partial x} \left( k_x \frac{\partial T}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_y \frac{\partial T}{\partial y} \right) + \frac{\partial}{\partial z} \left( k_z \frac{\partial T}{\partial z} \right) = 0 \quad (10)$$

where  $k_x$ ,  $k_y$  and  $k_z$  are, respectively, the thermal conductivity along  $x$ ,  $y$  and  $z$  directions. By setting different boundary conditions of the cubic cell, the heat flux in the cell can be controlled. As shown in Fig. 3, the up and down boundaries of the cell are at uniform, and constant temperature and the other boundaries are insulated. In this case, the heat flux is along the  $z$  direction. These boundary conditions are described as follows:

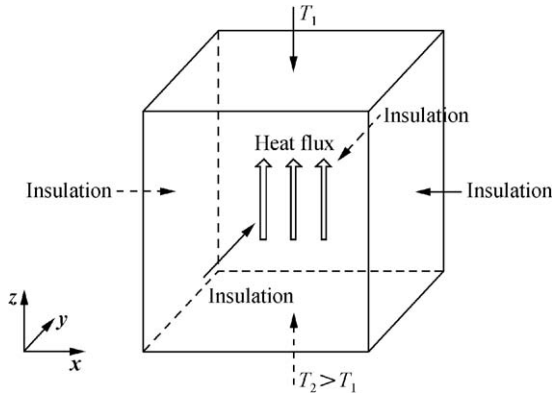


Fig. 3. Model of heat conduction in magnetic fluid.

$$T|_{z=0} = T_2, T|_{z=L} = T_1 \quad (11)$$

$$\left. \frac{\partial T}{\partial x} \right|_{y=0} = \left. \frac{\partial T}{\partial x} \right|_{y=L} = \left. \frac{\partial T}{\partial y} \right|_{x=0} = \left. \frac{\partial T}{\partial y} \right|_{x=L} = 0 \quad (12)$$

Three-dimensional uniform grids are generated in the cell and finite-difference equations are obtained by using the energy balance method. The heat flux from grid  $(i-1, j, k)$  to grid  $(i, j, k)$  can be expressed as

$$q_x = \frac{k_{i,j,k}^{x-} (T_{i-1,j,k} - T_{i,j,k})}{\Delta x} \quad (13)$$

As shown in Fig. 4, the heat fluxes to grid  $(i, j, k)$  from its other adjoining grids are  $q_{x+\Delta x}$ ,  $q_y$ ,  $q_{y+\Delta y}$ ,  $q_z$ ,  $q_{z+\Delta z}$ .

The energy balance equation for node  $(i, j, k)$  is expressed as

$$q_x + q_{x+\Delta x} + q_y + q_{y+\Delta y} + q_z + q_{z+\Delta z} = 0 \quad (14)$$

Substituting the heat flux expression into Eq. (12), we obtain

$$T_{i,j,k} = \frac{k_{i,j,k}^{x-} T_{i-1,j,k} + k_{i,j,k}^{x+} T_{i+1,j,k} + k_{i,j,k}^{y-} T_{i,j-1,k} + k_{i,j,k}^{y+} T_{i,j+1,k} + k_{i,j,k}^{z-} T_{i,j,k-1} + k_{i,j,k}^{z+} T_{i,j,k+1}}{k_{i,j,k}^{x-} + k_{i,j,k}^{x+} + k_{i,j,k}^{y-} + k_{i,j,k}^{y+} + k_{i,j,k}^{z-} + k_{i,j,k}^{z+}} \quad (15)$$

where  $k_{i,j,k}^{x-}$ ,  $k_{i,j,k}^{x+}$ ,  $k_{i,j,k}^{y-}$ ,  $k_{i,j,k}^{y+}$ ,  $k_{i,j,k}^{z-}$ ,  $k_{i,j,k}^{z+}$  are the effective thermal conductivities on the interfaces of nodal region  $(i, j, k)$ . The effective thermal conductivity  $k$  on the interface of two neighboring grids is calculated by the harmonic mean:

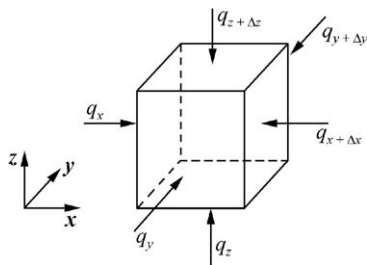


Fig. 4. Conduction into a grid from its adjoining grids.

$$k = \frac{2k_1 k_2}{k_1 + k_2} \quad (16)$$

where  $k_1$  and  $k_2$  are, respectively, the thermal conductivities of two neighboring grids. We use the SOR iteration method to solve the finite-difference equations. After the temperature distribution is obtained, the heat flux through every grid can be determined by Fourier's law. The total heat flux  $Q_z$  through any cross section can be obtained by summing the grid heat flux included in the cross section. Using Fourier's law, the thermal conductivity along the  $z$  direction is obtained as

$$k_{ez} = \frac{Q_z}{L(T_1 - T_2)} \quad (17)$$

By changing boundary conditions of the cell, namely, changing the heat flux direction, we can get the thermal conductivity along the  $x$  direction  $k_{ex}$  and that along the  $y$  direction  $k_{ey}$ .

### 3.2. Thermal conductivity of grid

When we generate 3D uniform grids in the cell, the grids can be classified into three types: (1) the grid is filled with base liquid, so the thermal conductivity of the grid equals the base liquid's thermal conductivity; (2) the grid is filled with part of a particle, so the thermal conductivity of the grid equals the particle material's thermal conductivity; (3) the grid is filled with both base liquid and part of a particle, the thermal conductivity of the grid can be determined by the following two methods.

#### 3.2.1. Method I

We calculate the thermal conductivity of the third type grid by the weighted average of the base liquid's thermal conductivity and the particle material's thermal conductivity according to the volume ratio of the base liquid and the particle in the grid.

#### 3.2.2. Method II

Using the method mentioned in Section 3.1, we can get the thermal conductivities of the third type grid along  $x$ ,  $y$  and  $z$  directions. Because the third type grid consisted of base liquid and part of a particle, when we generate sub-grids in the third type grid, the sub-grids also can be classified into three types. We use Method I to compute the third type sub-grids' thermal conductivity. Therefore, Method II takes into account the anisotropic heat conduction in the local grid region. Its precision is higher than that

of Method I. In the following calculations we use Method II to calculate the third type grid's thermal conductivity.

### 3.3. Precision analysis

To estimate the validity of the present method, the thermal conductivities of magnetic fluids with a small particle volume fraction are calculated. The results are compared with those of the Maxwell formula (Table 1). In the computation, the temperatures of the up and down boundaries are set to be  $T_1 = 300$  K and  $T_2 = 500$  K, respectively. At the room temperature  $T = 293$  K, the thermal conductivities of particle material and base liquid are  $k_p = 6.0$  W/mK and  $k_f = 0.6$  W/mK, respectively.

The Maxwell formula is expressed as [14]

$$\frac{k_e}{k_f} = \frac{k_p + 2k_f - 2\phi(k_f - k_p)}{k_p + 2k_f + \phi(k_f - k_p)} \quad (18)$$

Table 1 shows that the results of the two methods are in good coincidence. It indicates that the present method has a high precision in calculating the effective thermal conductivity of magnetic fluid.

### 3.4. Results and discussions

The thermal conductivity of magnetic fluid both parallel to the magnetic field direction ( $z$  direction) and perpendicular to the magnetic field direction ( $x$  and  $y$  directions) are calculated. Fig. 5(a) shows the thermal conductivities of magnetic fluids with different volume fractions in the absence of an external magnetic field. It is obvious that the thermal conductivity of the magnetic fluid is larger than that of the pure base liquid. And the thermal conductivity of the magnetic fluid increases with the increase of the particle volume fraction. The reason is that the particles have larger thermal conductivity than pure base liquid. The suspended particles in the base liquid enhance the thermal transport in the fluid, which leads to the magnetic fluids having larger thermal conductivity than pure base liquid. For a higher particle volume fraction, there are more particles suspended in the base liquid. Hence, the magnetic fluid has a higher thermal conductivity. Furthermore, the thermal conductivities along different directions are the same. The reason is that in the absence of an external magnetic field, although some dispersive clusters are formed in the magnetic fluid, the distribution of particles is isotropic.

Table 1

Thermal conductivity of magnetic fluid with uniform particle distribution.

Particle volume fraction (%)	Maxwell formula (W/mK)	Present method (W/mK)	Relative error (%)
1	0.6136	0.6139	5
2	0.6274	0.6278	6
3	0.6414	0.6420	9
4	0.6557	0.6565	12
5	0.6701	0.6711	15

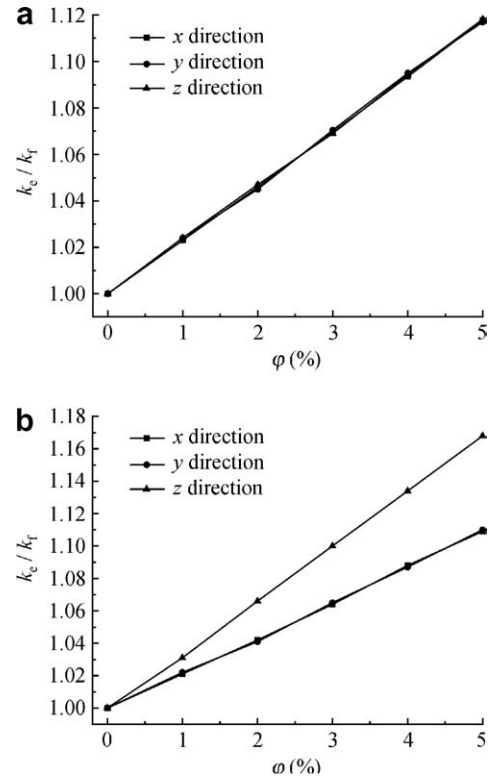


Fig. 5. Thermal conductivity of magnetic fluid in the absence of an external magnetic field (a) and in the presence of an external magnetic field (b).

Therefore, the thermal conductivity of the magnetic fluid is isotropic.

Fig. 5(b) shows the thermal conductivities of magnetic fluid along different directions in the presence of a strong magnetic field ( $\lambda_h = 20$ ). It is obvious that the thermal conductivities along  $x$  and  $y$  directions are the same, and both smaller than that along  $z$  direction. The thermal conductivity presents an anisotropic feature. With the increase of the magnetic field strength, the anisotropic feature of the thermal conductivity becomes more evident. The reason is that in the presence of a strong magnetic field, the particles form chainlike structures along the magnetic field. The particle chains provide more effective bridges for the thermal transport process along the direction of the magnetic field. With increasing particle volume fraction, the number of particles suspended in the fluid increases and more chains appear in the magnetic fluid, and the thermal conductivity along the magnetic field further increases. Hence, the anisotropy of the thermal conductivity becomes more evident.

Fig. 6(a) illustrates the thermal conductivity of a magnetic fluid (along  $x$  direction) with different particle volume fractions. A little change in the thermal conductivity is found with increasing magnetic field strength. Although in the presence of an external magnetic field, the particles form chainlike structures in the magnetic fluid, the particle chains affect weakly the thermal transport in the magnetic fluid perpendicular to the chain direction.

Fig. 6(b) shows the thermal conductivity of the magnetic fluid along the magnetic field direction with different

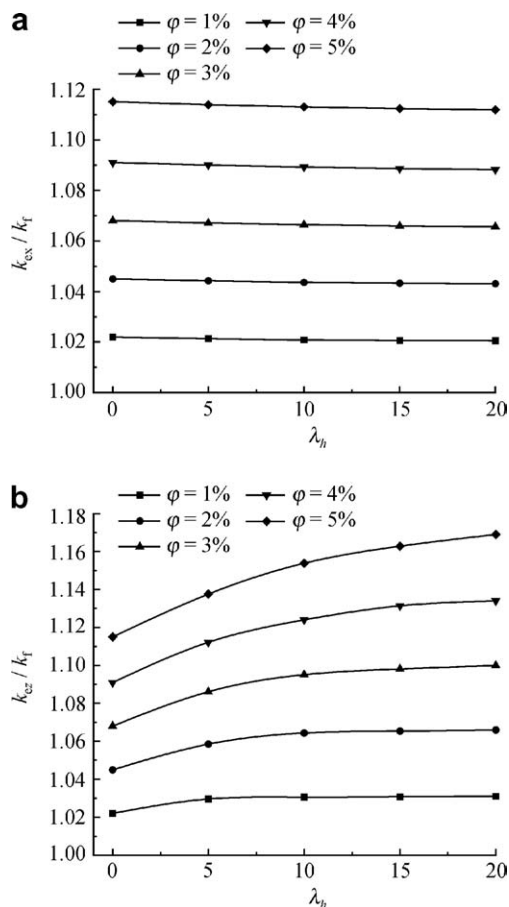


Fig. 6. Thermal conductivity of magnetic fluid perpendicular to the magnetic field direction (x direction) (a) and along the magnetic field direction (z direction) (b).

particle volume fractions. It is obvious that the thermal conductivity of the magnetic fluid increases with the increase of the magnetic field strength. This is because that with the increase of the magnetic field strength, the magnetization of the magnetic fluid increases and the chainlike structures in the magnetic fluid become more obvious. The particle chains provide more effective bridges for thermal transport along the direction of the magnetic field. When the magnetization of the magnetic fluids gets close to the saturation, a little change in the chainlike structures occurs and a little thermal conductivity increment is found with the further increase of magnetic field strength.

In a word, the morphology of magnetic particles suspended in base liquid controls the energy transport inside the magnetic fluid. Therefore, it is possible to control the heat transfer process inside the magnetic fluid through changing the aggregation structure of particles by applying an external magnetic field.

#### 4. The modification of the Maxwell formula

The analysis above indicates that the morphology of magnetic particles suspended in base liquid controls the energy transport inside the magnetic fluid. In fact, in the

presence of an external magnetic field, the distribution of particles in the magnetic fluid is not uniform. The particle volume fraction is a spatial function. Taking into account the non-uniform distribution of particles, we introduce an anisotropic structure parameter to modify the particle volume fraction of magnetic fluids and extend the traditional Maxwell formula to calculate the anisotropic thermal conductivity of the magnetic fluid. The main process is as follow: By accounting for the relative ubiquity between every two particles in the simulation system, and then taking the ensemble average, the anisotropic structure parameter in different directions can be obtained. For example, the anisotropic structure parameter in the x direction ( $C_x$ ) is expressed as

$$C_x = \frac{1}{N} \sum_{i=1}^N \sum_{j>i}^N C_{ij}^x \quad (19)$$

$$C_{ij}^x = \frac{(\mathbf{n}_x \cdot \mathbf{e}_{ij})^2 - 1/4}{(r_{ij}/d)^3} \quad (20)$$

where subscripts  $i$  and  $j$  are particle numbers,  $d$  is the diameter of the particle,  $\mathbf{r}_{ij} = \mathbf{r}_i - \mathbf{r}_j$ ,  $r_{ij} = |\mathbf{r}_{ij}|$ ,  $\mathbf{e}_{ij} = \mathbf{r}_{ij}/r_{ij}$ ,  $\mathbf{r}_i$  is the position of the  $i$ th particle,  $\mathbf{n}_x$  is the unit vector along the x direction.

In Eqs. (19) and (20), we can find that when the particle volume fraction increases, the distance between two particles  $r_{ij}$  decreases, the value of the anisotropic structure parameter  $C_x$  gets larger. And when the chainlike structure along the x direction becomes more obvious, the value of the anisotropic structure parameter  $C_x$  gets larger. The anisotropic structure parameter  $C_x$  well characterizes the distribution of particles along the x direction. By taking into account the anisotropic structure parameter, the modified particle volume fraction is defined as  $\phi_x = (1 + C_x)\phi$ . Substituting the expression of  $\phi_x$  into Eq. (18), we obtain

$$\frac{k_{ex}}{k_f} = \frac{k_p + 2k_f - 2\phi_x(k_f - k_p)}{k_p + 2k_f + \phi_x(k_f - k_p)} \quad (21)$$

Eq. (21) is capable of calculating the thermal conductivity of magnetic fluid along the x direction. We can use the same method to calculate the thermal conductivity of magnetic fluid along the y and z directions.

The microstructures of magnetic fluids with different particle volume fractions have been obtained in the second part of this paper. By modifying the particle volume fraction and substituting the modified volume fraction into Eq. (21), we calculate the anisotropic thermal conductivity of the magnetic fluids. Fig. 7 demonstrates the comparison of the anisotropic thermal conductivity of magnetic fluids calculated by the modified Maxwell formula (real line) and those computed by the numerical method mentioned in the third part of this paper (dashed line). The comparison shows that the results of the two methods are in good agreement. It indicates that the modified Maxwell formula can be used to calculate the anisotropic thermal conductivity of magnetic fluids with non-uniform particle distribution.

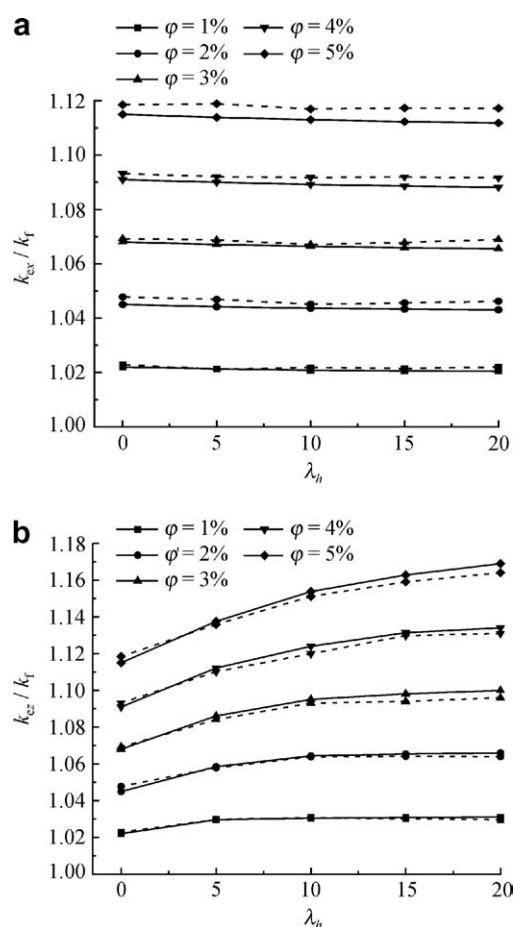


Fig. 7. Comparison of the two methods for thermal conductivity. (a) Perpendicular to the magnetic field (x direction); (b) parallel to the magnetic field (z direction).

## 5. Conclusions

The microstructure of magnetic fluid under the influence of an external magnetic field has been investigated by the numerical simulation method and the anisotropic heat conduction in the magnetic fluid has been analyzed. A numerical method for predicting the anisotropic thermal conductivity of magnetic fluid has been proposed.

By introducing an anisotropic structure parameter which characterizes the non-uniform distribution of particles in the magnetic fluid and modifying the particle volume fraction involved in Maxwell formula, the traditional Maxwell formula is modified and extended to calculate anisotropic thermal conductivity of the magnetic fluid.

The aggregation structure of magnetic particles controls the heat conduction in the magnetic fluid. In the absence of an external magnetic field, the distribution of particles in the magnetic fluid is disordered and the thermal conductivity of magnetic fluid is isotropic. In the

presence of an external magnetic field, the particles form chainlike clusters along the magnetic field direction, which leads to an increment in the thermal conductivity along the chain direction and almost no change in the thermal conductivity perpendicular to the chain. The thermal conductivity presents anisotropic feature. With the increase of the magnetic field strength, the chainlike structure becomes more obvious and the anisotropic feature of heat conduction in the magnetic fluid becomes more evident. Furthermore, with increasing the particle volume fraction, more chains appear in the magnetic fluid and the anisotropic feature of heat conduction in the magnetic fluid becomes more evident.

## Acknowledgement

This work sponsored by National Natural Science Foundation of China (Grant No. 50436020).

## References

- [1] Li DC. Theories and applications of magnetic fluids. 1st ed. Beijing: Science Press; 2003, [in Chinese].
- [2] Liu JH, Gu JM. Thermodynamic properties and applications of magnetic fluids. *Journal of Functional Materials and Devices* 2002;8:3314–8.
- [3] Akira S, Chantrell RW, Shin-Ichi K, et al. Two-dimensional Monte Carlo simulations to capture thick chainlike clusters of ferromagnetic particles in colloidal dispersions. *J Colloid Interface Sci* 1996;178:2620–7.
- [4] Satoh A, Chantrell RW, Shin-Ichi, et al. Three dimension Monte Carlo simulations of thick chainlike clusters composed of ferromagnetic fine particles. *J Colloid Interface Sci* 1996;181:2422–8.
- [5] Satoh A, Chantrell RW, Coverdale GN. Brownian dynamics simulations of ferromagnetic colloidal dispersions in a simple shear flow. *J Colloid Interface Sci* 1999;209:44–59.
- [6] Yoshihisa E, Katsumi O, Masafumi O. Simulation study on microstructure formations in magnetic fluids. *Physica A* 2003;330:496–506.
- [7] Li Q, Xuan YM, Li B. Simulation and control scheme of microstructure in magnetic fluids. *Sci China (Ser D)* 2007;50:3371–9.
- [8] Kronkalns G. Measurement of the thermal and electrical conductivities of a ferrofluid in a magnetic field. *Magnitnaya Gidrodinamika* 1977;3:138–40.
- [9] Li Q, Xuan Y, Wang J. Experimental investigations on transport properties of magnetic fluids. *Experimental Thermal and Fluid Science* 2005;30:109–16.
- [10] Mohebi M, Jamasbi N, Liu J. Simulation of the formation of nonequilibrium structure in magnetorheological fluids subject to an external magnetic field. *Phys Rev E* 1996;54:5407–13.
- [11] Russel WB, Saville DA, Schowalter WR. *Colloidal dispersion*. Cambridge: Cambridge University Press; 1989.
- [12] Coverdale GN, Chantrell RW, Satoh A, et al. Molecular dynamic model of the magnetic properties and microstructure of advanced metal particle dispersions. *J Appl Phys* 1997;81:83818–20.
- [13] McQuarrie DA. *Statistical mechanics*. University of California, Davis: University Science Book; 2000.
- [14] Maxwell JC. *A treatise on electricity and magnetism*. 2nd ed. Oxford: Clarendon Press; 1881.